

# Strain Rate Self-Sensing for a Cantilevered Piezoelectric Beam

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This paper deals with the analytical modeling, and the experimental verification of the strain rate self-sensing method using a hybrid adaptive filter for a cantilevered piezoelectric beam. The piezoelectric beam consists of two laminated lead zirconium titanates (PZT) on a metal shim. A mathematical model of the beam dynamics is derived by Hamilton's principle and the accuracy of the modeling is verified through the comparison with experimental results. For the strain rate estimation of the cantilevered piezoelectric beam, a self-sensing mechanism using a hybrid adaptive filter is considered. The discrete parts of this mechanism are realized by the DS1103 DSP board manufactured by dSPACE™. The efficacy of this method is investigated through the comparison of experimental results with the predictions from the derived analytical model.

**Key Words :** Piezoelectric Material, Adaptive Filter, Strain Rate Self-Sensing, Hamilton's Principle

## Nomenclature

$A$  : Area of electrode  
 $b$  : Width of piezoelectric beam  
 $C_P$  : Equivalent capacitance of piezoelectric material  
 $D_3$  : Electric charge displacement (charge per unit area)  
 $d_{31}$  : Piezoelectric material constant  
 $e_{31}$  : Piezoelectric material constant  
 $E_3$  : Electric potential supplied in 3 - axis  
 $E_P^{11}$  : Young's modulus of piezoelectric material  
 $E_S$  : Young's modulus of center shim material  
 $E_C$  : Young's modulus of coating material  
 $h_P$  : Thickness of piezoelectric material  
 $h_S$  : Thickness of center shim material  
 $h_C$  : Thickness of coating material

$i_{tot}$  : Total current applied on the piezoelectric material  
 $i_{elec}$  : Feedthrough current on the piezoelectric material  
 $i_{mech}$  : Current used for mechanical works  
 $L$  : Length of piezoelectric beam  
 $m_S$  : Mass per unit length for shim material  
 $m_P$  : Mass per unit length for piezoelectric material  
 $m_C$  : Mass per unit length for coating material  
 $n$  : Total number of modes in assumed mode expansion  
 $Q_{tot}$  : Total charge applied on the piezoelectric material  
 $Q_{mech}$  : Equivalent charge due to the developed mechanical strain  
 $r_i(t)$  :  $i$ -th generalized coordinate  
 $v$  : Supplied voltage on the electrode  
 $\varepsilon_1$  : Mechanical strain in 1 - axis  
 $\varepsilon_{33}^S$  : Permittivity of piezoelectric material at constant strain  
 $\phi_i(x)$  :  $i$ -th mode shape function

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- $\rho_S$  : Density of shim material  
 $\rho_P$  : Density of piezoelectric material  
 $\rho_C$  : Density of coating material

## 1. Introduction

Due to its advantages in smaller size and weight, and higher bandwidth capabilities as compared to the other devices, the piezoelectric materials have been used extensively as sensors or actuators. The piezoelectric materials can act as a transducer, generating a voltage difference between the two electrodes in response to an imposed force, thus sensing dynamic acceleration, pressure, etc. (Doebelin, 1990). Conversely, the response of piezoelectric materials to the applied voltages allows them to be used as actuators in a wide variety of applications ranging from the vibration to the noise suppression problems (Dimitriadis et al., 1991; Clark and Fuller, 1992; Evans et al., 1999; Ko, 1996; Okugawa and Sasaki, 1997; Oh et al., 1998). However, the combined actuation and sensing mechanisms (i.e. the self-sensing capability) for the piezoelectric materials are possible and becoming very important for the intelligent mechanical structures. By implanting the piezoelectric materials functioning both as actuators and sensors in the composite materials, the intelligent structures can be made, which change their mechanical properties adaptable for the various operational environments, or which have self-monitoring capabilities of the mechanical structure's malfunction (Crawley and Luis, 1987; Ishihara and Arai, 1996).

The idea of self-sensing actuators was concurrently proposed by Dosch et al. (1992) and Anderson et al. (1992). By applying the identical voltages to a piezoelectric actuator and an equivalent capacitor, and comparing the currents from both, the strain rates of the mechanical structure can be estimated. This method, however, turns out to be impractical due to the changes of the piezoelectric capacitance with the operational conditions (Johns et al., 1994; Janocha et al., 1994). Clark and et. al. proposed a strain rate self-sensing mechanism using the least-mean-square

(LMS) algorithm to resolve the problems of the previous method (Cole and Clark, 1994). They also proposed a hybrid adaptive filter mechanism using the steepest descent algorithm to estimate the strain rate of piezoelectric structure (Vipperman and Clark, 1998). A continuous-time adaptive filter for estimating the strain rate was introduced by Fannin and Saunders (1997).

This paper deals with the analytical modeling and the experimental verification of the strain rate self-sensing method using a hybrid adaptive filter for a cantilevered piezoelectric beam. The piezoelectric beam consists of two laminated lead zirconium titanates (PZT) on a metal shim. Section 2 contains the introduction and working principles of the various self-sensing methods. In Sec. 3, the mathematical model of the cantilevered piezoelectric beam dynamics is derived by Hamilton's principle and the accuracy of the modeling is verified through the comparison with experimental results. Section 4 includes the experimental results from the strain rate self-sensing system. For the strain rate estimation of the cantilevered piezoelectric beam, a self-sensing mechanism using a hybrid adaptive filter is considered. The discrete parts of this mechanism are realized by the DS1103 DSP board manufactured by dSpaceTM. The efficacy of this method is investigated through the comparison of experimental results with those from the analytical method. The effect of convergence factor in the adaptive filter is also experimentally investigated. Finally, the conclusion of this paper and future research areas are summarized in Sec. 5.

## 2. Methods of Strain Rate Self-Sensing

Basically, the piezoelectric material is an energy transformer, which converts the electrical energy to the mechanical energy, and vice versa. When it is used for an actuation device, only a portion of the supplied electric energy is converted to the useful mechanical energy. The rest is fed through the piezoelectric material, and eventually dissipated as heat in the electric circuits. Figure 1

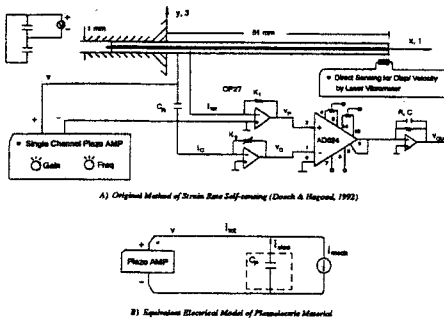


Fig. 1 Original method of strain rate self-sensing

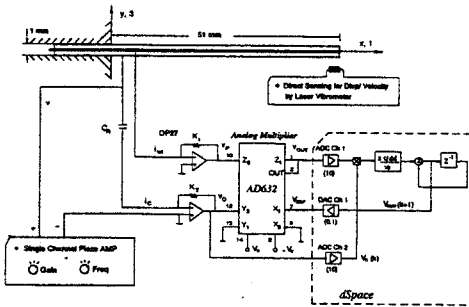


Fig. 2 Method of strain rate self-sensing using adaptive filter

(b) conceptually shows this phenomenon. It shows that only a portion ( $i_{mech}$ ) of the total current ( $i_{tot}$ ) is transformed into the useful mechanical energy. A self-sensing mechanism is a kind of estimator for this mechanical energy. Two methods of strain rate self-sensing are considered here. The circuit in Fig. 1(a) is the original method proposed by Anderson et al., and Dosch et al., in 1992. Figure 2 shows a circuit using a method developed by Clark (1995 ; 1998) employing an adaptive filter for a strain rate estimation.

**2.1 The original method (Hagood and Dosch, 1992)**

Figure 1(a) represents the original circuit for a strain rate sensing proposed by Hagood and Dosch in 1992. The one-dimensional constitutive equation for the piezoelectric beam is given by the following.

$$D_3 = e_{31}\epsilon_1 + \epsilon_{33}^s E_3$$

$$v = E_3 h_P = \frac{h_P}{\epsilon_{33}^s} (D_3 - e_{31}\epsilon_1) \tag{1}$$

The last relation of Eq. (1) can be rearranged further by introducing the electrode area,  $A$ .

$$v = \frac{h_P}{A\epsilon_{33}^s} (AD_3 - Ae_{31}\epsilon_1) = \frac{1}{C_P} (q_{tot} - q_{mech}) \tag{2}$$

where,  $q_{tot} = D_3 A$   
 $q_{mech} = e_{31} A \epsilon_1$   
 $C_P = \frac{\epsilon_{33}^s A}{h_P}$

By differentiating of Eq. (2), the following current relation can be established.

$$i_{tot} = i_{elec} + i_{mech} = C_P \frac{dv}{dt} + i_{mech} \tag{3}$$

The Eq. (2) and (3) are the basic equations for a self-sensing mechanism. By measuring electrically either  $q_{mech}$  or  $i_{mech}$ , the mechanical strain, or its rate developed on a piezoelectric material can be estimated.

The electrical circuit of Fig. 1(a) is based on the Eq. (3). The reference capacitor,  $C_R$  is used to cancel the feed through current. The total supplied current ( $i_{tot}$ ) and feed through current ( $i_{elec}$ ) are converted into the voltage,  $v_P$  and  $v_C$ , and compared to each other using a high CMMR differential amplifier (Analog Device, AD524). Because of noise, a 1- st-order LPF (Low Pass Filter) with the cutoff frequency of 5 kHz is applied at the final stage. The output voltage of AD524, which has a unity gain, is given by

$$v_s = v_P - v_C = (-K_1 i_{tot}) - \left( -K_2 C_R \frac{dv}{dt} \right)$$

$$= -K_1 i_{mech} + \frac{dv}{dt} (K_2 C_R - K_1 C_P) \tag{4}$$

The term inside the last parenthesis in Eq. (4) can be vanished by adjusting the variable resistor,  $K_2$ . After adjustment, the output voltage of the LPF is

$$v_{OUT} = \frac{K_1 i_{mech}}{1 + sRC} \cong K_1 e_{31} A \frac{d\epsilon_1}{dt} \tag{5}$$

If the value of  $C_P$  does not change during the operation of the piezoelectric material, the above method works perfectly. However, it is known that the value of a piezoelectric capacitance ( $C_P$ )

varies with the magnitude ( $q$ ) and frequency ( $\omega$ ) of driving signal as well as with the operational temperature ( $T$ ), and even with time ( $t$ ).

$$C_P = f(q, \omega, T, t) \quad (6)$$

## 2.2 The method of adaptive filtering (Clark and Vipperman, 1995)

This method was developed by Clark and Vipperman in 1995. The original method of a strain self-sensing described in the above section works properly only under the stationary operating conditions for the piezoelectric material. If the operating conditions change, the recalibration of  $K_2$  is necessary. Clark and Vipperman proposed a self-sensing mechanism using an adaptive filter, which obviates the troublesome manual tuning of  $K_2$ . Their concept is depicted in Fig. 2, which is mechanized using dSPACE™ DS1103 board. The AD632 IC in Fig. 2 is a high CMMR input analog multiplier which has the following transfer function.

$$\begin{aligned} v_{OUT} &= \frac{(X_1 - X_2)(Y_1 - Y_2)}{10} + Z_2 \\ &= v_P - \frac{v_C V_{DSP}}{10} \\ &= -K_1 i_{tot} + \frac{K_2}{10} i_C V_{DSP} \\ &= -K_1 \left\{ i_{mech} + \left( C_P - \frac{K_2}{10K_1} C_R V_{DSP} \right) \frac{dv}{dt} \right\} \quad (7) \end{aligned}$$

The term inside the small parenthesis in the above equation is similar to that of Eq. (4). The value of  $v_{OUT}$  in Eq. (7) is minimized automatically by adjusting  $V_{DSP}$  through a recursive adaptive filter instead of manual tuning as was done in Eq. (4).

The value of  $V_{DSP}$  is determined by minimizing the quadratic cost function of the following equation.

$$J(V_{DSP}) = v_{OUT}^2 \quad (8)$$

From Eq. (7) and (8), the cost function,  $J(V_{DSP})$  is a quadratic form in  $V_{DSP}$ , which means that its global minimum uniquely exists. By using the steepest descent method, the direction of change of  $V_{DSP}$  is calculated as follows.

$$\frac{dV_{DSP}}{dt} = -\mu \nabla V_{DSP} J(V_{DSP}) \quad (9)$$

The term of  $\nabla V_{DSP} J(V_{DSP})$  in Eq. (9) physically means the increasing direction of the cost function,  $J(V_{DSP})$  for  $V_{DSP}$ . Therefore, the solution of this differential equation always gives the global minimum of  $J(V_{DSP})$ . And, the value of  $\mu$  controls the convergence speed to the global minimum. The above differential equation can be approximated by the following difference equation of

$$\begin{aligned} \frac{dV_{DSP}}{dt} &\approx \frac{V_{DSP}(k+1) - V_{DSP}(k)}{\Delta t} \\ &= -2\mu v_{OUT}(k) \frac{\partial v_{OUT}}{\partial V_{DSP}} \quad (10) \end{aligned}$$

Therefore,  $V_{DSP}(k)$  can be determined by the 1-st order recursive digital filter of

$$V_{DSP}(k+1) = V_{DSP}(k) + \frac{2\mu\Delta t}{10} v_{OUT}(k) v_C(k) \quad (11)$$

If  $V_{DSP}(k)$  converges, then the capacitance of a piezoelectric material can be estimated by

$$C_P = \frac{K_2}{10K_1} C_R V_{DSP} \quad (12)$$

The difference equation of Eq. (11) is the main equation in the strain rate self-sensing using an adaptive filter algorithm. This filter dynamics are mechanized on the dSPACE™ 1103 DSP board as shown in Fig. 2.

## 3. Mathematical Model of a Cantilevered Piezoelectric Beam

The piezoelectric beam used in this paper is manufactured by the FUJI Ceramics (Model #: BM-ACT 03). A schematic diagram of the piezoelectric beam is shown in Fig. 3. It is composed of a central shim (Fe and Ni alloy) between two pieces of PZT materials with a coating material (RTV Silicone). The material properties are summarized in Table 1. From the assumption of an Euler-Bernoulli beam theory, only the bending stress in the cantilevered beam exists in the  $x$ -direction. Therefore, the displacement vector,  $U(X, t)$  for the beam motion can be simplified as a scalar function,  $y(x, t)$ , which can be approximated using the assumed mode method.

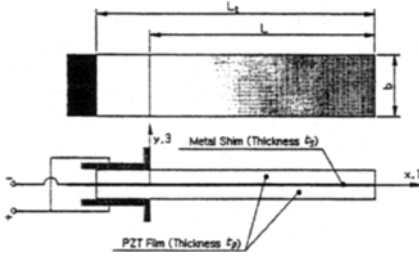


Fig. 3 Schematics of the cantilevered piezoelectric beam

$$U(X, t) = y(x, t) \cong \sum_{i=1}^n \phi_i(x) r_i(t) \quad (13)$$

where,  $\phi_i(x)$  : Admissible function satisfying geometric boundary condition

$r(t) = [r_1(t) \ \cdots \ r_n(t)]^T$  : Generalized coordinate vector

A mathematic model governing the motion of the piezoelectric cantilevered beam can be derived using Hamilton's principle and assumed mode method as follows.

$$M_s \ddot{r} + C_D \dot{r} + K_s r = \Theta v \quad (14)$$

$$q = \Theta^T r + C_P v$$

where,

$M_s$  : Mass matrix associated with center shim, piezoelectric and coating material ( $n \times n$ )

$$M_s = [m_{ij}]$$

$$= \left[ (m_s + 2m_p + 2m_c) \int_0^L \phi_i(x) \phi_j(x) dx \right] \quad (15)$$

$$m_s = \rho_s h_s b$$

$$m_p = \rho_p h_p b$$

$$m_c = \rho_c h_c b$$

$K_s$  : Stiffness matrix associated with center shim, piezoelectric and coating material ( $n \times n$ )

$$K_s = [k_{ij}] = \left[ (E_s I_s + 2E_p I_p + 2E_c I_c) \int_0^L \phi_i''(x) \phi_j''(x) dx \right] \quad (16)$$

$$I_s = \frac{b h_s^3}{12}$$

$$I_p = \frac{b}{3} \left( h_p^3 + \frac{3}{2} h_p^2 h_s + \frac{3}{4} h_p h_s^2 \right)$$

$$I_c = \frac{b}{3} \left\{ 3 \left( h_p + \frac{h_s}{2} \right)^2 h_c + 3 \left( h_p + \frac{h_s}{2} \right) h_c^2 + h_c^3 \right\}$$

$C_D$  : Damping matrix of the total structure ( $n \times n$ )

$r$  : Generalized mechanical coordinate ( $n \times 1$ )  
 $\Theta$  : Electromechanical coupling matrix ( $n \times m$ )

$$\Theta = \begin{pmatrix} -a \int_0^L \phi_1''(x) dx \\ \vdots \\ -a \int_0^L \phi_n''(x) dx \end{pmatrix} \quad (17)$$

$$\alpha = \frac{e_{31} b}{h_p} \int_{h_s/2}^{h_s/2+h_p} y dy = \frac{e_{31} b}{2} (h_s + h_p)$$

$$= \frac{d_{31} E_p^{11} b}{2} (h_s + h_p)$$

$v$  : Generalized electrical coordinate ( $1 \times 1$ )

$q$  : Vector of applied electrode charge ( $1 \times 1$ )

$C_P$  : Diagonal matrix of piezoelectric capacitance ( $1 \times 1$ )

$$C_P = 2 \frac{\epsilon_{33} b L \epsilon_0}{h_p} \quad (18)$$

It's difficult to analytically include the effect of damping on the beam motion. Therefore, the damping matrix  $C_D$  of Eq. (14) is developed through the coordinate transformation between the original and modal axis. Appropriate modal dampings are found from the comparison of analytic and experimental results.

To calculate the structural matrix of  $M_s$ ,  $K_s$ , and  $\Theta$  in Eq. (14), the following assumed mode, which is an admissible function satisfying the geometric boundary conditions of cantilevered beam, is used. For the approximation of  $y(x, t) \cong \sum_{i=1}^n \phi_i(x) r_i(t)$ , two admissible functions are used, i.e.

$$\phi_i(x) = \frac{x}{L} \sin\left(\frac{\pi x}{2L} i\right) \quad (19)$$

$$\phi_i(x) = \cosh\left(\frac{\pi x}{2L} i\right) - \cos\left(\frac{\pi x}{2L} i\right)$$

$$- \frac{\sinh(\pi i/2) - \sin(\pi i/2)}{\cosh(\pi i/2) + \cos(\pi i/2)}$$

$$\left( \sinh\left(\frac{\pi x}{2L} i\right) - \sin\left(\frac{\pi x}{2L} i\right) \right) \quad (20)$$

The admissible function of Eq. (20) is closer to the eigenfunction of a cantilevered beam problem than the one of Eq. (19). However, it turns out that this difference of admissible functions is not a factor, as there are no significant effects on the

**Table 1** Characteristic data of the piezoelectric beam

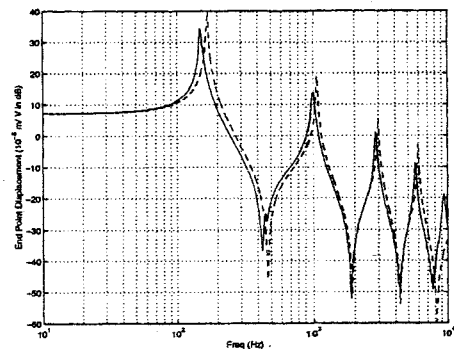
Symbol	Description	Value	Dimension
$L$	Clamped length of piezoelectric beam	51	mm
$L_t$	Total length of piezoelectric beam	68	mm
$h_P$	Thickness of piezoelectric beam	0.45	mm
$h_s$	Thickness of elastic beam (Shim)	0.1	mm
$h_c$	Thickness of coating material (RTV silicone)	0.02	mm
$b$	Width of beam	10	mm
$\rho_P$	Density of piezoelectric material	$7.75 \times 10^3$	kg/m <sup>3</sup>
$\rho_s$	Density of shim material	$8.2 \times 10^3$	kg/m <sup>3</sup>
$\rho_c$	Density of coating material	$1.16 \times 10^3$	kg/m <sup>3</sup>
$E_P^{11}$	Young's modulus of piezoelectric material	$5.9 \times 10^{10}$	N/m <sup>2</sup>
$E_s$	Young's modulus of shim material	$1.47 \times 10^{11}$	N/m <sup>2</sup>
$E_c$	Young's modulus of coating material	$6.2 \times 10^8$	N/m <sup>2</sup>
$d_{31}$	Piezoelectric constant	$-330 \times 10^{-12}$	m/V
$\epsilon_{33}^r$	Relative permittivity of piezoelectric material	5500	F/m
$C_P$	Total capacitance of piezoelectric material	109.6	nF

resulting structural matrix of Eq. (14).

To evaluate the accuracy of the developed mathematical model, the experimental frequency response is found. The experimental setup for this work is exactly the same as that depicted in Fig. 2 except that the frequency response analyzer (ONO SOKKI, CF350 Dual Channel FFT Analyzer) drives the cantilevered piezoelectric beam. The end point displacement of the piezoelectric beam is picked up by the laser displacement sensor (Keyence LD-2500 (10 mV/ $\mu$ m sensitivity)). The analytical and experimental frequency response of the beam end point are plotted together in Fig. 4. The modal damping of 0.01 is used for all the modal frequencies. The dashed line is the analytical result and the other is for the experimental result. The analytical frequency response is shifted up 4.4 dB to match DC gain with the experimental results in this figure. The analytical frequency response is almost the same as that of experimental one except that it is shifted to the right on the frequency axis. The comparison of modal frequencies is shown on Table 2. Approximately, 5% relative error between the analytic and experimental modal frequency is resulted. The analytic and experimental step responses of the beam end point for 10 Volt driving are shown in Fig. 5. The lower one is the analytic response, which has the DC gain of about 0.675 (-3.4 dB) times less than that of the

**Table 2** Comparison of analytic and experimental modal frequencies

Modal Frequency	Experiment (Hz)	Analysis (Hz)	% Error
1 - st mode	160	168.5	5.31
2 - nd mode	1025	1059	3.31
3 - rd mode	2850	2988	4.84
4 - th mode	5625	5962	5.99

**Fig. 4** The frequency response of beam end point displacement (analytical and experimental)

experimental one.

#### 4. Experiments on Strain Rate Self-Sensing Using Adaptive Filter

The resistor value of  $K_2$  in Fig. 2 is chosen as  $2 \text{ k}\Omega$  in order for  $V_{DSP}$  to be within linear opera-

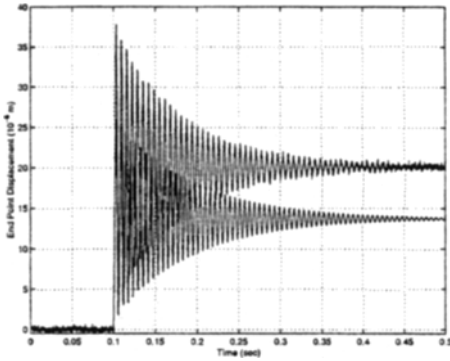


Fig. 5 The analytical and experimental step responses for 10 V driving signal

tion range. The use of smaller value of  $K_2$  like  $1 \text{ k}\Omega$  makes the magnitude of  $V_{DSP}$  to be at around 10 Volt due to the relation of Eq. (7), which is near the saturation range of the dSPACE™ 1103 DSP board. It turns out the convergence of  $V_{DSP}$  is not guaranteed for some driving frequencies in case of using  $1 \text{ k}\Omega$  for  $K_2$ . The experimental frequency response of  $v_{OUT}$  for the excitation signal of  $v$  is shown as the solid line in Fig. 6 with the analytic result, which is the dashed line. The sampling time of the digital adaptive filter is  $100 \text{ }\mu\text{s}$ , and the convergence factor ( $\mu$ ) is set as a 400 in this experiment. There are two points that should be mentioned in the comparison with two responses. One is the DC level difference. The other, which is much more critical, is that the experimental frequency response does not show the effects of zero. The main reason for these is considered due to the hysteresis phenomenon of the piezoelectric materials. Because the piezoelectric materials are ferroelectric, they fundamentally exhibit hysteresis behavior in their response to an applied electric field (Ping and Jouaneh, 1996). The hysteresis loop is a consequence of the effects of domain switching due to the action of the external applied cyclic electric field. Since each domain is a composition of several parallel dipoles, the effective number of dipoles aligned in the direction of the electric field will be altered as domains switch under the action of an external electric field. This domain switching does not occur instantaneously and it is this delay response which gives rise to the

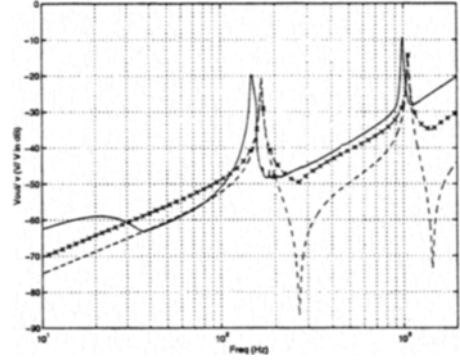


Fig. 6 The frequency response of  $v_{OUT}$  from adaptive filter

hysteresis loop. Therefore, the current flow ( $i_{tot}$ ) through the piezoelectric material has the phase delay compared to the current flow ( $i_c$ ) through the reference capacitance in Fig. 2.

In order to see the effect of the phase delay from a piezoelectric material on the performance of a strain rate self-sensing circuit, it's assumed that  $i_{mech}=0$ . This assumption is almost true for the excitation at any zero frequencies. Then, for the excitation input of  $v=v_0 \sin \omega t$ ,

$$\begin{aligned}
 v_{out} &= v_p - \frac{v_c V_{DSP}}{10} = -K_1 C_p \left( \frac{dv}{dt} \right)_p \\
 &\quad - \frac{K_2}{10} C_r V_{DSP} \left( \frac{dv}{dt} \right)_c \\
 &= -v_0 \omega K_1 C_p \cos(\omega t - \phi) \\
 &\quad + \frac{v_0 \omega K_2 C_r}{10} V_{DSP} \cos \omega t \\
 &\cong -\frac{v_0 \omega K_2 C_r \phi}{10} V_{DSP} \sin \omega t \\
 &= -v_0 \omega K_1 C_p \phi \sin \omega t
 \end{aligned} \tag{21}$$

In the above derivation, the perfect functioning of the adaptive filter is assumed, i.e.

$$V_{DSP} = \frac{10 C_p K_1}{C_r K_2} \tag{22}$$

The validity of the above analysis can be evaluated by the investigation of the experimental frequency response in Fig. 6. The analytical frequency response of  $v_{OUT}$ , which is calculated based on the assumption of the phase delay of  $1^\circ$ , is shown as the dash-dot line with 'x' mark in this figure. The disappearance of zero effects due to the phase delay in the frequency response can be noticed. If the frequency information can be extracted from the driving signal in real-time, the

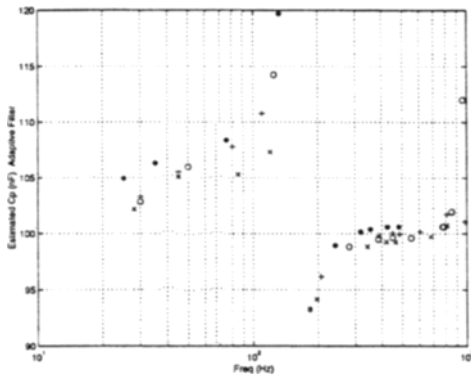


Fig. 7 The estimated capacitance of piezoelectric material

phase delay effects can be cancelled out from the adaptive filtering.

The capacitance of the piezoelectric material is estimated based on the experimental data on the self-sensing circuit using adaptive filtering. The manufacturer (FUJI Ceramics) supplied data of the piezoelectric capacitance has a value of 109.6 nF. Eq. (12) can be used to estimate the capacitance of piezoelectric material. The experimental results are shown in Fig. 7. The points marked as ‘\*’, ‘+’, ‘o’, and ‘x’ in this figure show the values of the estimated capacitance at each excitation frequency for the driving signal magnitudes of 20.5  $V_{PP}$ , 12.5  $V_{PP}$ , 8.75  $V_{PP}$ , and 5.5  $V_{PP}$ , respectively. Based on these data, the estimated capacitance of the piezoelectric material seems to be affected by the cantilever beam motion dynamics.

The effect of convergence factor,  $\mu$  in Eq. (9) can be noticed in Fig. 8 and Fig. 9. The experimental data for  $V_{DSP}$ ,  $v_{OUT}$ , and  $v_C$  are shown in Fig. 8 for the value of  $\mu=400$ , and in Fig. 9 for that of  $\mu=2$ . These data are obtained by varying the excitation signal frequency while keeping its magnitude constant at 10  $V_{PP}$ . The signal frequency is varied continuously from 200 Hz to 800 Hz, and back to the 200 Hz using a function generator. The magnitude of  $v_{OUT}$  is much smaller than that of  $v_C$ , indicating that the adaptive filter is functioning in the strain rate self-sensing circuit. The slow convergence of  $V_{DSP}$  in Fig. 9 is because of using the small value of convergence factor,  $\mu$ . The peak-to-peak value of

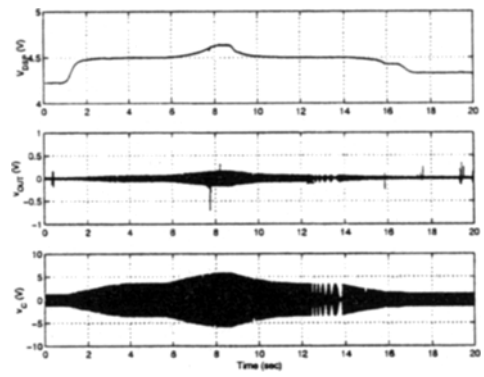


Fig. 8 The variation of  $V_{DSP}$ ,  $v_{OUT}$ , and  $v_C$  for sweeping sine input ( $\mu= 400$ )

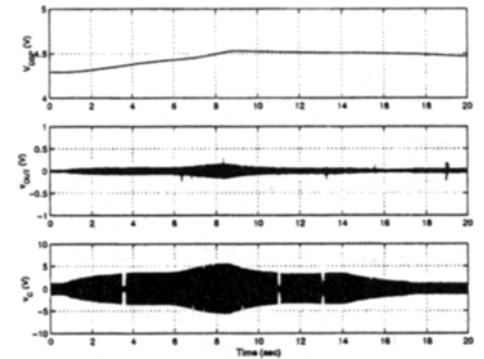


Fig. 9 The variation of  $V_{DSP}$ ,  $v_{OUT}$ , and  $v_C$  for sweeping sine input ( $\mu=2$ )

$v_{OUT}$  at the final stage (around 20 seconds in Fig. 9) becomes larger than the initial value by the factor of two due to the slow convergence of  $V_{DSP}$ . The peak-to-peak values of  $v_{OUT}$  at the final stage for  $\mu=400$ , however, are almost same order of magnitude at the initial and final stage, as can be seen in Fig. 8.

The several observations can be made based on the analysis of the experimental results. The strain rate self-sensing mechanism using an adaptive filtering (Vipperman, 1995 and Clark, 1998), which is applied to the problem of a cantilevered piezoelectric beam, does not work properly in the points that the zero frequency dynamics of the beam motion are not possible to identify experimentally and the estimated capacitance of the piezoelectric material (or  $V_{DSP}$  at the steady state, by referring Eq. (12)) changes its value according to the variation of the beam dynamics.



The main reason of disappearing zero dynamics is considered as the phase delay in the piezoelectric material. And, the variation of the estimated piezoelectric capacitance with the beam motion is supposed to be the byproduct of using an adaptive filtering structure. This can be explained by the further analysis on Eq. (7) - Eq. (9).

$$\begin{aligned} \frac{dV_{DSP}}{dt} &= -\mu \nabla V_{DSP} J(V_{DSP}) \\ &= 2\mu \frac{v_c v_{out}}{10} = \frac{2\mu}{10} K_1 K_2 C_r \left( \frac{dv}{dt} \right) \\ &\left\{ \left( C_p - \frac{K_2}{10K_1} C_r V_{DSP} \right) \left( \frac{dv}{dt} \right) + i_{mech} \right\} \end{aligned} \quad (23)$$

As can be seen in the above equation,  $V_{DSP}$  at the steady state is dependant on the magnitude of  $i_{mech}$ . Therefore, the magnitude of  $V_{DSP}$  would have the relatively larger value at the pole frequencies of the cantilevered beam dynamics than that at the other frequencies. This reasoning is supported by the experimental results of Fig. 8, in which the large values of the piezoelectric capacitance (or  $V_{DSP}$ ) are noticed at the modal frequencies of 160 Hz and 1025 Hz.

## 5. Conclusion

The problem of the strain rate self-sensing is considered both analytically and experimentally for a cantilevered PZT beam. The self-sensing structure is implemented using a hybrid adaptive filter based on the steepest descent algorithm. A mathematical model is derived by the application of Hamilton's principle to the cantilevered piezoelectric beam. The major discrepancies between the experimental and analytical results of the strain rate self-sensing are the magnitude differences near the zero frequencies of the cantilevered beam dynamics. It is believed that these are due to the phase delay effect of the piezoelectric material. It turns out also that the estimated capacitance of the piezoelectric material is affected by the cantilever beam motion dynamics. This is considered to be the natural consequence in using an adaptive filtering for a strain rate self-sensing. The future research areas are focused on finding the new and systematic self-sensing mechanism to estimate the strain rate

variation of the piezoelectric structure and the estimator design of the beam end point displacement from the strain rate self-sensing information.

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